Year 12 Semester 1 Examination, 2016

Question/Answer Booklet

Hale School

MATHEMATICS SPECIALIST

Section Two
Calculator Assumed

Student Name

Teacher: (circle) Mr Hill Mr Lau

Score: (out of 99)

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	12	12	100	99	65
			Total	152	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

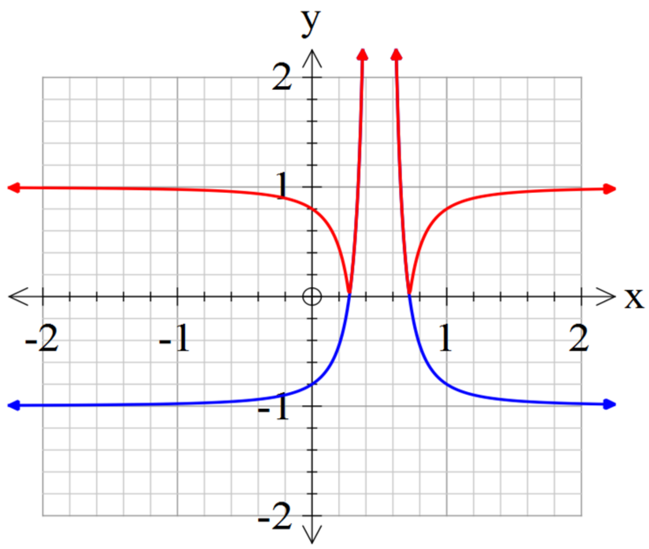
Section Two: Calculator-assumed **(99 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

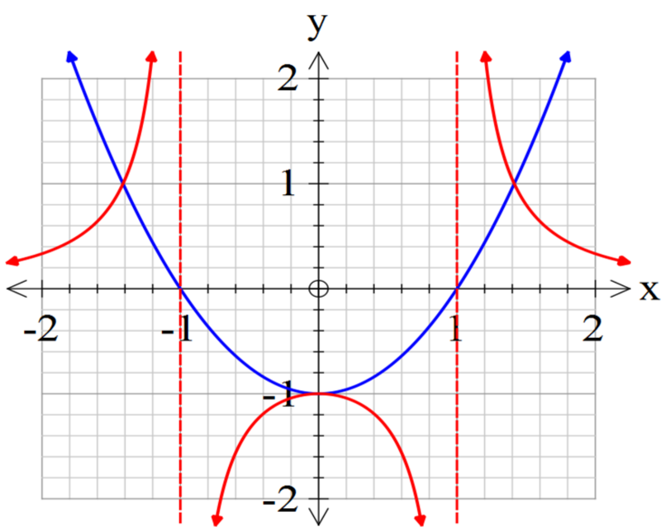
Question 8 **(5 marks)**

(a) Given the graph of $y = f(x)$ below, sketch on the same axes the graph of $y = |-f(x)|$ (2 marks)



Correct shape ✓✓

(b) Given the graph of $y = f(x)$ below, sketch on the same axes the graph of $y = \dfrac{1}{f(x)}$. (3 marks)



Poles sketched ✓

Correct shape -1<x<1 ✓

Correct Shape x>1, x<-1 ✓

Question 9 **(8 marks)**

The line l_1 has the equation $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$.

(a) Find the acute angle between l_1 and the line joining the points P$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and Q$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$, giving your answer correct to the nearest degree. (3 marks)

$$\text{Direction of line}: \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}}{\sqrt{1^2 + 2^2 + 4^2}\sqrt{1^2 + (-5)^2}}$$

$$\cos\theta = \frac{-19}{\sqrt{21}\sqrt{26}}$$

$$\theta = 144.40°$$

$$180° - 144.40° = 36°$$

Direction of line ✓
Correct formula ✓
Final Answer ✓

(b) Determine the position vector of the point R that lies on the line joining P$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and Q$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$, such that PR : RQ = 1 : 2. (3 marks)

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \frac{1}{3}\begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

$$\overrightarrow{OR} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 4/3 \\ -1 \\ -2/3 \end{pmatrix}$$

States PQ ✓
Correct expression OR ✓
Correct Solution ✓

Question 9 continued…

(c) Find an equation of the plane through Q$\begin{pmatrix}2\\-1\\-4\end{pmatrix}$ and perpendicular to l_1, in the form $\underset{\sim}{r}\cdot\underset{\sim}{n}=\rho$.

(2 marks)

$\underset{\sim}{r}.\underset{\sim}{n}=\underset{\sim}{a}.\underset{\sim}{n}$

$\underset{\sim}{r}.\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}2\\-1\\-4\end{pmatrix}.\begin{pmatrix}1\\2\\4\end{pmatrix}$

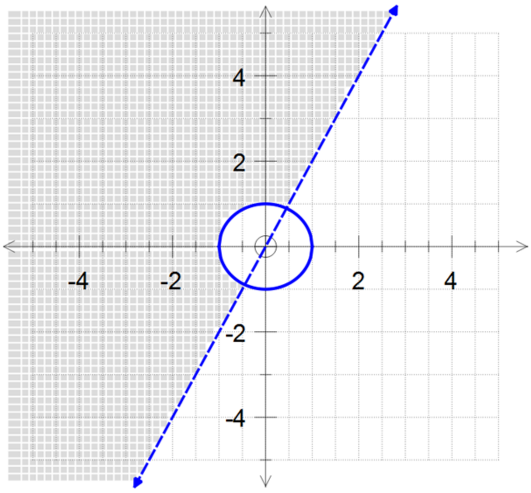
$\underset{\sim}{r}.\begin{pmatrix}1\\2\\4\end{pmatrix}=-16$

Correct Normal Vector ✓
Correct Constant ✓

Question 10 **(9 marks)**

Draw sketches of the following loci on the Argand diagrams provided.

(a) $\{z : |z| \geq 1 \text{ and } 2\,\text{Re}(z) < \text{Im}(z)\}$ (3 marks)

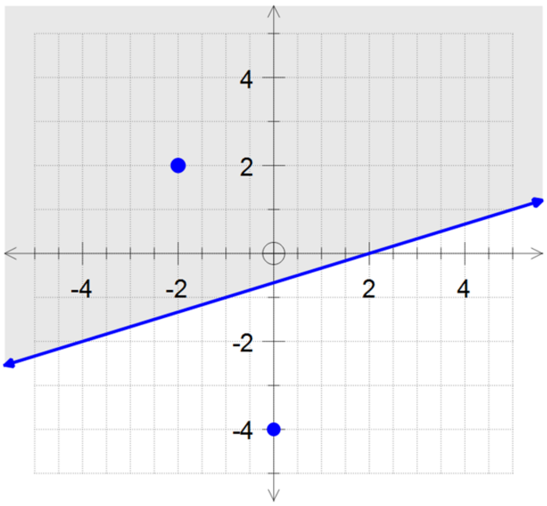


y>2x Sketched correctly ✓

Circle Correct ✓

Shading ✓

(b) $\{z : |z + 2 - 2i| \leq |z + 4i|\}$ (3 marks)

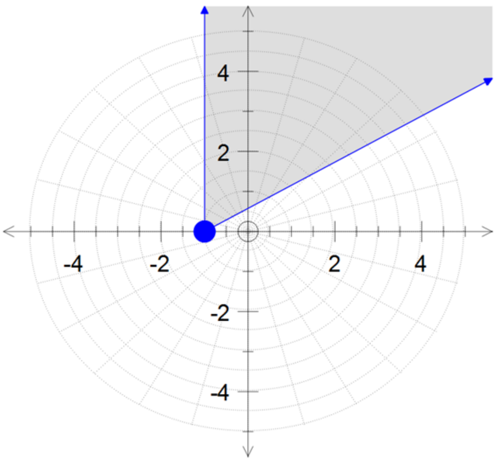


Correctly identifies (-2,2) (0,-4) ✓

Line correct ✓

Shading ✓

(c) $\left\{z : \frac{\pi}{6} \leq Arg(z+1) \leq \frac{\pi}{2}\right\}$ (3 marks)



Draws correct argument ✓

Correct translation ✓

Shading ✓

Question 11 **(7 marks)**

Let $\alpha = -1 + i$.

(a) Express α in polar form. (1 mark)

$$\alpha = \sqrt{2}cis(\frac{3\pi}{4})$$

Correct Answer ✓

(b) Show that α is a root of the equation $z^4 + 4 = 0$. (3 marks)

$$\left(\sqrt{2}cis(\frac{3\pi}{4})\right)^4 + 4$$
$$= \sqrt{2}^4 cis(4 \times \frac{3\pi}{4}) + 4$$
$$= 4cis3\pi + 4$$
$$= 4(-1) + 4$$
$$= 0$$

Uses de Moivre's Theorem ✓
Evaluates $cis3\pi$ correctly ✓
LHS=RHS ✓

(c) Hence or otherwise find a real quadratic factor of the polynomial $z^4 + 4$ (3 marks)

$$\alpha_1 = -1 + i \qquad \alpha_2 = -1 - i$$
$$\left(x - (-1+i)\right)\left(x - (-1-i)\right)$$
$$= x^2 - \left[(-1+i) + (-1-i)\right]x + (-1+i)(-1-i)$$
$$= x^2 + 2x + 2$$

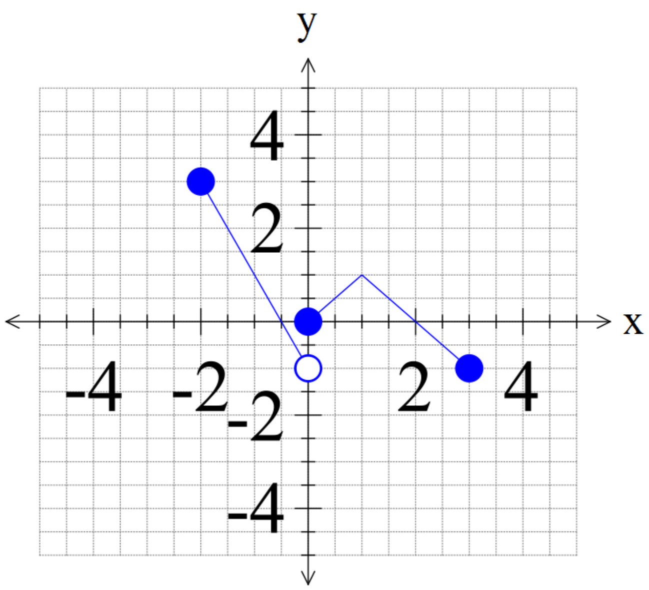
States Complex Conjugate pair ✓
Expands correctly ✓
States solution ✓

Question 12 **(9 marks)**

(a) Sketch the graph of $f(x)$ described by:

$$f(x) = \begin{cases} |2x| - 1 & -2 \le x < 0 \\ 1 - |x-1| & 0 \le x \le 3 \end{cases}$$

(3 marks)



Correct x<0	✓
Correct x>0	✓
Open circle x=0	✓

(b) Hence, or otherwise, determine $\int_{-2}^{3} f(x)\,dx$. (2 marks)

$$Sum\ \text{of}\ Area = \frac{1}{2}3\times1.5 - \frac{1}{2}0.5\times1 + 1\times1 - \frac{1}{2}1\times1$$
$$= 2.5$$

Uses Area under curve to calculate integral	✓
Correct answer	✓

(c) Use an algebraic approach to solve for k if $|1-2k| \ge 4|k+3|$. (4 marks)

• When k<-3

$1-2k \ge 4(-k-3)$

$-2k+4k \ge -1-12$

$k \ge -13/2$

• When -3<k<0.5

$1-2k \ge 4(k+3)$

$-2k-4k \ge -1+12$

$k \le -11/6$

• When k>0.5

$-1+2k \ge 4(k+3)$

$2k-4k \ge 1+12$

$k \le -13/2$

$\therefore -13/2 \le k \le -11/6$

Separates domain correctly	✓
Establishes correct inequalities	✓
Solves inequalities correctly	✓
Correct Answer	✓

Question 13 **(4 marks)**

If $z = \cos\theta + i\sin\theta$, show that $\tan\theta = \dfrac{z - z^{-1}}{i(z + z^{-1})}$.

$$RHS = \frac{\cos\theta + i\sin\theta - \dfrac{1}{\cos\theta + i\sin\theta}}{i\left(\cos\theta + i\sin\theta + \dfrac{1}{\cos\theta + i\sin\theta}\right)}$$

$$= \frac{\cos\theta + i\sin\theta - \dfrac{\cos\theta - i\sin\theta}{\cos^2\theta + i\sin^2\theta}}{i\left(\cos\theta + i\sin\theta + \dfrac{\cos\theta - i\sin\theta}{\cos^2\theta + i\sin^2\theta}\right)}$$

$$= \frac{\cos\theta + i\sin\theta - \cos\theta + i\sin\theta}{i(\cos\theta + i\sin\theta + \cos\theta - i\sin\theta)}$$

$$= \frac{2i\sin\theta}{2i\cos\theta}$$

$$= \tan\theta$$

$$= LHS$$

Multiplies by Complex Conjugate	✓
Uses $\cos^2 x + \sin^2 x = 1$	✓
Simplifies correctly	✓
LHS=RHS	✓

Question 14 **(15 marks)**

A particle moves such that its position from O at any time, $t > 0$ is given by

$$\boldsymbol{r}(t) = (\cos t + t.\sin t)\,\boldsymbol{i} + (\sin t - t.\cos t)\,\boldsymbol{j}$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the East and North direction respectively.

(a) Find, in terms of t and in simplest form,

(i) the distance of the particle from O, (3 marks)

$$\text{distance}=\sqrt{(\cos t + t\sin t)^2 + (\sin t - t\cos t)^2}$$
$$=\sqrt{\cos^2 t + 2t\cos t\sin t + t^2\sin^2 t + \sin^2 t - 2t\sin t\cos t + t^2\cos^2 t}$$
$$=\sqrt{1+t^2}\quad \textit{units}$$

States distance formula	✓
Expands correctly	✓
Simplifies	✓

(ii) the speed of the particle. (3 marks)

$$\dot{\boldsymbol{r}}(t) = (-\sin t + \sin t + t\cos t)\boldsymbol{i} + (\cos t - \cos t + t\sin t)\boldsymbol{j}$$
$$\dot{\boldsymbol{r}}(t) = t\cos t\,\boldsymbol{i} + t\sin t\,\boldsymbol{j}$$

$$\text{Speed}=\sqrt{t^2\cos^2 t + t^2\sin^2 t}$$
$$=\sqrt{t^2(\cos^2 t + \sin^2 t)}$$
$$=t \quad \textit{units / time}$$

Differentiates	✓
Calculates Speed correctly	✓
Simplifies	✓

(b) Find, correct to 3 decimal places, the first non-zero time when the particle is due East or due West of O. (2 marks)

$$\boldsymbol{r}_j \text{ comp } = 0$$

$$\sin t - t\cos t = 0$$
$$t = 4.493 \text{ units (CAS)}$$

$\boldsymbol{r}_j$ comp $= 0$	✓
Correct solution	✓

Question 14 continued…

(c) (i) Find an expression for ***a.r*** in simplest form and use it to find when ***a.r*** = 0.

(5 marks)

$\dot{\boldsymbol{r}}(t) = t\cos t\,\boldsymbol{i} + t\sin t\,\boldsymbol{j}$

$\ddot{\boldsymbol{r}}(t) = (\cos t - t\sin t)\boldsymbol{i} + (\sin t + t\cos t)\boldsymbol{j}$

$\boldsymbol{a.r} = \left((\cos t - t\sin t)\boldsymbol{i} + (\sin t + t\cos t)\boldsymbol{j}\right).\left((\cos t + t.\sin t)\boldsymbol{i} + (\sin t - t.\cos t)\boldsymbol{j}\right)$

$= \cos^2 t - t^2\sin^2 t + \sin^2 t - t^2\cos^2 t$

$= 1 - t^2$

$\boldsymbol{a.r} = 0$

$1 - t^2 = 0$

$(1-t)(1+t) = 0$

$t = 1, \cancel{-1}$

Determines Acceleration	✓
Expands Dot Product	✓
Simplifies expression	✓
Factorises	✓
Finds Solution	✓

(ii) Interpret the result from (c) (i) in terms of the motion of the particle.

(2 marks)

At $t = 1$ the acceleration is perpendicular to the position vector. ✓

Change in velocity is tangential to its position. ✓

Question 15 **(11 marks)**

(a) The line $\underset{\sim}{r} = \begin{pmatrix} 4+3\lambda \\ 1-3\lambda \\ 1+\lambda \end{pmatrix}$ is inclined to the plane $\underset{\sim}{r}.\begin{pmatrix} 1 \\ a \\ -2 \end{pmatrix} = 4$ at an angle of 30° .

Find a . (4 marks)

The angle between the normal vector of the plane and the direction vector of the line must be 90-30=60°

$$\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}.\begin{pmatrix} 1 \\ a \\ -2 \end{pmatrix} = \sqrt{3^2+(-3)^2+1^2}\sqrt{1^2+a^2+(-2)^2}\cos 60$$

$$1-3a = \sqrt{19}\sqrt{5+a^2} \times \frac{1}{2}$$

$$a = -1.713 \quad or \quad 3.12 \quad \text{(CAS)}$$

Determines Correct Angle	✓
Applies correct formula	✓
Simplifies	✓
Solution	✓

(b) Find the vector equation of the plane with Cartesian equation $2x - y = 3 - z$. (1 mark)

$$\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 3$$

✓

Question 15 continued…

(c) Does the line $\underset{\sim}{r} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k} + \lambda(3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$ lie on the plane $\underset{\sim}{r}.(\underset{\sim}{i} + 6\underset{\sim}{j} + 3\underset{\sim}{k}) = 17$? Justify your answer. (2 marks)

$$\boldsymbol{r}.\begin{pmatrix}1\\6\\3\end{pmatrix}$$
$$= \begin{pmatrix}2+3\lambda\\3-\lambda\\-1+\lambda\end{pmatrix}.\begin{pmatrix}1\\6\\3\end{pmatrix}$$
$$= 2 + 3\lambda + 18 - 6\lambda - 3 + 3\lambda$$
$$= 17$$

$\therefore$ lies on plane

Substitutes line into plane	✓
Shows result	✓

(d) Determine the equation of the plane which is parallel but 5 units away from the plane $\underset{\sim}{r}.\begin{pmatrix}3\\2\\-2\end{pmatrix} = 4$ (4 marks)

Let new plane Π: $\boldsymbol{r}.\begin{pmatrix}3\\2\\-2\end{pmatrix} = x$

A point on the original plane $= \begin{pmatrix}0\\2\\0\end{pmatrix}$

A line $\perp$ to original plane: $\boldsymbol{r} = \begin{pmatrix}0\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\-2\end{pmatrix}$

The intersection of the line and Π

$$\begin{pmatrix}3\lambda\\2+2\lambda\\-2\lambda\end{pmatrix}.\begin{pmatrix}3\\2\\-2\end{pmatrix} = x$$
$$9\lambda + 4 + 4\lambda + 4\lambda = x$$
$$\lambda = \frac{x-4}{17}$$

$$\lambda\left|\begin{pmatrix}3\\2\\-2\end{pmatrix}\right| = \pm 5$$
$$\frac{x-4}{17}\sqrt{17} = \pm 5$$
$$x = 4 \pm 5\sqrt{17}$$

$$\therefore \Pi:\ \boldsymbol{r}.\begin{pmatrix}3\\2\\-2\end{pmatrix} = 4 \pm 5\sqrt{17}$$

Recognises new plane shares normal vector	✓
Determines eqn line $\perp$ to plane	✓
Determines value(s) for x where distance = 5	✓
Determines equation(s) of a plane	✓

Question 16 **(16 marks)**

A particle A is projected with velocity $12\mathbf{i}+(24-10t)\mathbf{j}$ from the origin O, where *t* seconds is the time after projection. At the same time, particle B is projected from $80\mathbf{i}+5\mathbf{j}$ so that its displacement is given by $(80-20t)\mathbf{i}+(5+22t-5t^2)\mathbf{j}$. All distances are given in metres.

(a) Which particle was projected with the greatest initial speed, and what was this speed?

(4 marks)

$$\text{Speed of A} = |\boldsymbol{v}_A| = \sqrt{720}$$
$$\text{Vel of B} = -20\boldsymbol{i}+(22-10t)\boldsymbol{j}$$
$$\text{Speed of B} = |\boldsymbol{v}_B| = \sqrt{884}$$
$$\text{B has greatest speed } 29.73\ m/s$$

Calculates Speed A	✓
Differentiates to get vel of B	✓
Calculates Speed B	✓
Evaluates greatest speed	✓

(b) How far apart are A and B after 1 second? (5 marks)

$$\boldsymbol{r}_A(\text{t}) = 12t\boldsymbol{i}+(24t-5t^2)\boldsymbol{j}+\boldsymbol{c}$$
$$when\ t=0, \boldsymbol{c}=0$$
$$\boldsymbol{r}_A(\text{t}) = 12t\boldsymbol{i}+(24t-5t^2)\boldsymbol{j}$$
$$\boldsymbol{r}_A(1) = 12\boldsymbol{i}+19\boldsymbol{j}$$

$$\boldsymbol{r}_B(1) = 60\boldsymbol{i}+22\boldsymbol{j}$$

$$dist = \sqrt{(60-12)^2+(22-19)^2}$$
$$= 48.09\ m$$

Integrates v_A correctly	✓
Determines $r_A(1)$	✓
Determines $r_B(1)$	✓
Correct expression for distance	✓
Correct Solution	✓

Question 16 continued…

(c) Which particle reached the greatest height, and how much higher was this than the other particle? (3 marks)

$(v_A)j = 0$

$24 - 10t = 0$

$t = 2.4\text{s}$ Height A = 28.8 m

$(v_B)j = 0$

$22 - 10t = 0$

$t = 2.2\text{s}$ Height B = 29.2 m

B reaches greatest height by 0.4 m

Determines Height A	✓
Determines Height B	✓
Correct difference	✓

(d) Show that the particles collide and state the time and position of the collision. (4 marks)

If the particles collide

$$\begin{pmatrix} 12t \\ 24t - 5t^2 \end{pmatrix} = \begin{pmatrix} 80 - 20t \\ 5 + 22t - 5t^2 \end{pmatrix}$$

$i: \quad 12t = 80 - 20t$

$t = 2.5s$

$j: \quad 24t - 5t^2 = 5 + 22t - 5t^2$

$t = 2.5s$

$\therefore$ Particles collide after 2.5s at the position $30\boldsymbol{i} + 28.75\boldsymbol{j}$

Equates position vectors	✓
Evaluates t for i and j	✓✓
Correct statement and position	✓

Question 17 **(5 marks)**

Suppose that a complex number w, lies on the unit circle, and $0 \le \arg(w) \le \frac{\pi}{2}$.

Prove that $2\arg(w+1) = \arg w$.

$$2\arg(w+1) = \arg w \qquad \text{let } w = x + yi$$

$$2\tan^{-1}\left(\frac{y}{x+1}\right) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\tan\left(2\tan^{-1}\left(\frac{y}{x+1}\right)\right) = \tan\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$$

$$\tan\left(2\tan^{-1}\left(\frac{y}{x+1}\right)\right) = \frac{y}{x}$$

$$LHS = \tan\left(2\tan^{-1}\left(\frac{y}{x+1}\right)\right)$$

$$= \frac{2\left(\frac{y}{x+1}\right)}{1-\left(\frac{y}{x+1}\right)^2}$$

$$= \frac{\frac{2y}{x+1}}{\frac{(x+1)^2 - y^2}{(x+1)^2}}$$

$$= \frac{2y(x+1)}{(x+1)^2 - y^2}$$

$$= \frac{2y(x+1)}{(x^2+2x+1)-(1-x^2)} \qquad \left(x^2 + y^2 = 1 \text{ - Unit Circle}\right)$$

$$= \frac{2y(x+1)}{2x^2+2x}$$

$$= \frac{2y(x+1)}{2x(x+1)}$$

$$= \frac{y}{x}$$

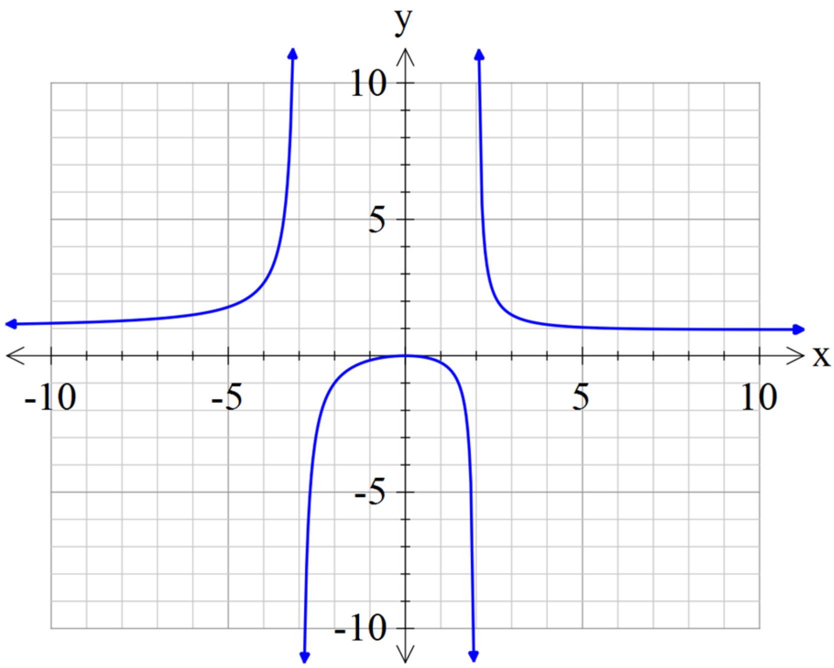
$$= RHS$$

- Takes tan of both sides ✓
- Uses $\tan 2x$ identity ✓
- Simplifies fraction ✓
- Recognises Unit Circle $x^2 + y^2 = 1$ ✓
- Factorises Correctly ✓

Question 18 **(4 marks)**

Sketch the graph of the rational function $f(x)$ given that it has the following properties:

- $f(0)=0, f'(0)=0$
- $\lim_{x\to -3^-} f(x) = \lim_{x\to 2^+} f(x) \to \infty$
- $\lim_{x\to -3^+} f(x) = \lim_{x\to 2^-} f(x) \to -\infty$
- $\lim_{x\to -\infty} f(x) = \lim_{x\to \infty} f(x) = 1$



Correct poles ✓

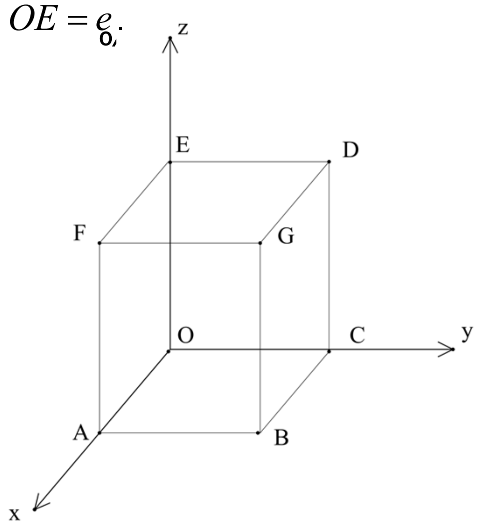
T.P. at (0,0) ✓

Correct shape -3<x<2 ✓

Correct shape -3>x x>2 ✓

Question 19 **(6 marks)**

OABCDEFG is a rectangular prism with $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{OE} = \underset{\sim}{e}$.



(a) Using the fact that $|\underset{\sim}{a} \times \underset{\sim}{c}| = |\underset{\sim}{a}||\underset{\sim}{c}|\sin\theta$, show that the area of the base OABC is given by Area $= |\underset{\sim}{a} \times \underset{\sim}{c}|$.

(2 marks)

$$\begin{aligned} \text{Area} &= l \times w \\ &= |\boldsymbol{a}| \times |\boldsymbol{c}| \\ &= |\boldsymbol{a}| \times |\boldsymbol{c}| \sin 90^\circ \qquad (\angle OAC = 90^\circ) \\ &= |\boldsymbol{a} \times \boldsymbol{c}| \end{aligned}$$

Expresses length and width as magnitude of vectors	✓
Shows sine angle	✓

(b) Prove that the volume of the rectangular prism is given by $(\underset{\sim}{a} \times \underset{\sim}{c}) \cdot \underset{\sim}{e}$. (4 marks)

Area base $= |\boldsymbol{a} \times \boldsymbol{c}|$

height $= |\boldsymbol{e}|$

Volume $= |\boldsymbol{a} \times \boldsymbol{c}||\boldsymbol{e}|$

$\boldsymbol{a} \times \boldsymbol{c}$ is parallel to $\boldsymbol{e}$ $\quad \therefore$ angle is 0

$$\cos(0) = \frac{(\boldsymbol{a} \times \boldsymbol{c}).\boldsymbol{e}}{|\boldsymbol{a} \times \boldsymbol{c}||\boldsymbol{e}|}$$

$$|\boldsymbol{a} \times \boldsymbol{c}||\boldsymbol{e}| = (\boldsymbol{a} \times \boldsymbol{c}).\boldsymbol{e}$$

Expression for Volume	✓
Expresses Parallelism	✓
Uses Dot product	✓
Conclusion	✓

Additional working space

Question number: _________

Additional working space

Question number: _________